Answers to exam-style questions

Topic 6

Where appropriate, 1 ✓ = 1 mark

1 A

2 C

3 B

4 C

5 C

6 B

7 D

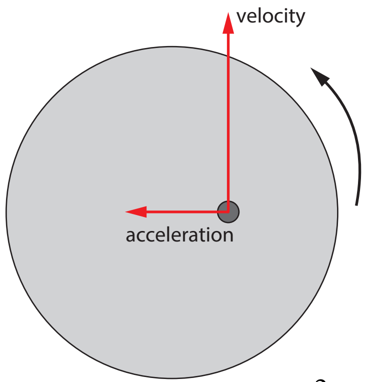
8 D

9 C

10 A

11 **a** Velocity arrow. ✓

Acceleration arrow. ✓



b The angular speed is $\omega = \dfrac{2\pi}{1.40} = 4.488 \approx 4.5\ \text{rad s}^{-1}$. ✓

The linear speed is $v = \omega r = 4.488 \times 0.22 = 0.987 \approx 0.99\ \text{m s}^{-1}$. ✓

c At maximum distance the frictional force will be the largest possible, i.e. $f_{\text{max}} = \mu_s N = \mu_s mg (= 0.434\ \text{N})$. ✓

$\mu_s mg = m\dfrac{v^2}{r} = m\dfrac{\omega^2 r^2}{r}$, hence $r = \dfrac{\mu_s g}{\omega^2}$ ✓

$r = \dfrac{0.82 \times 9.8}{4.488^2} = 0.399 \approx 0.40\ \text{m}$ ✓

d **i** Using $r = \dfrac{\mu_s g}{\omega^2}$ we find $\omega = \sqrt{\dfrac{\mu_s g}{r}}$ ✓

$\omega = \sqrt{\dfrac{0.82 \times 9.8}{0.22}} = 6.0\ \text{rad s}^{-1}$ ✓

ii The static frictional force can no longer supply the larger centripetal force required. ✓

The body will then slide and the static frictional force is now replaced by the even smaller sliding frictional force; hence the disc will slide off the rotating platform. ✓

12 a From energy conservation: $\frac{1}{2}mv^2 = mgL$ so $v = \sqrt{2gL}$, ✓

$v = \sqrt{2 \times 9.8 \times 2.0} = 6.26 \approx 6.3\ \text{m s}^{-1}$. ✓

b $a = \frac{v^2}{L} = \frac{6.26^2}{2.0} = 19.6 \approx 20\ \text{m s}^{-2}$. ✓

c Weight vertically downwards. ✓

Larger arrow for tension upwards. ✓

d i A particle is in equilibrium if it moves with constant velocity. ✓

This particle moves on a circle and so cannot be in equilibrium. ✓

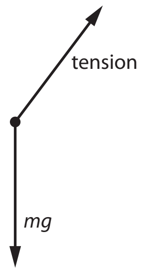
ii $T - mg = \frac{mv^2}{L}$ ✓

$T = \frac{mv^2}{L} + mg = \frac{5.0 \times 6.26^2}{2.0} + 5.0 \times 9.8 = 147 \approx 150\ \text{N}$ ✓

(or better: $T = \frac{mv^2}{L} + mg = \frac{m \times 2gL}{L} + mg = 3mg = 3 \times 5.0 \times 9.8 = 147 \approx 150\ \text{N}$)

13 a Correct arrows for tension. ✓

Correct arrow for weight. ✓



b A particle is in equilibrium if it moves with constant velocity. ✓

This particle moves on a circle and so cannot be in equilibrium. ✓

c i The vertical component of the tension equals the weight and so $T\cos\theta = mg$, i.e. $T = \frac{mg}{\cos\theta}$. ✓

The horizontal component of the tension is $T\sin\theta$ and $T\sin\theta = m\frac{v^2}{r} = m\frac{v^2}{L\sin\theta}$ ✓

Combining gives the answer $v = \sqrt{\frac{gL\sin^2\theta}{\cos\theta}}$.

ii The angular and linear speeds are related by $v = \omega r = \omega L\sin\theta$. ✓

So $\omega = \frac{\sqrt{\frac{gL\sin^2\theta}{\cos\theta}}}{L\sin\theta}$. ✓

Which is the answer $\omega = \sqrt{\frac{g}{L\cos\theta}}$.

d i $v = \sqrt{\frac{9.8 \times 0.45 \times \sin^2 60°}{\cos 60°}} = 2.57 \approx 2.6\ \text{m s}^{-1}$ ✓

ii $\theta = \sqrt{\frac{9.8}{0.45 \times \cos 60°}} = 6.5997 \approx 6.6\ \text{rad s}^{-1}$ ✓

e i The air resistance force will reduce the speed of the ball. ✓

ii A graph of $\frac{\sin^2\theta}{\cos\theta}$ shows that because the speed decreases, the angle will also decrease. ✓

iii The cosine of the angle will increase and hence the angular speed will decrease. ✓

(**Note:** These questions are best answered by considering the total energy of the ball:

$$E = \frac{1}{2}mv^2 + mgh = \frac{1}{2}m\frac{gL\sin^2\theta}{\cos\theta} + mgL(1-\cos\theta) = \frac{1}{2}mgL\left(\frac{\sin^2\theta + 2\cos\theta - 2\cos^2\theta}{\cos\theta}\right)$$

The air resistance will reduce the total energy; graphing the total energy as a function of angle θ shows that for the energy to decrease the angle must decrease.)

14 a Measuring distances from the top of the sphere and using energy conservation shows that:

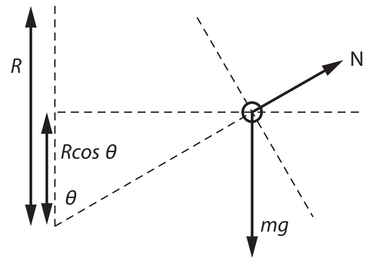
$0 = \frac{1}{2}mv^2 - mgh$ where h is the vertical distance the marble falls. ✓

From trigonometry: $h = R(1-\cos\theta)$. ✓ (see diagram that follows in **b**)

And so $0 = \frac{1}{2}mv^2 - mgR(1-\cos\theta)$. ✓

Manipulating gives $v = \sqrt{2gR(1-\cos\theta)}$.

b The forces on the marble are the weight mg and the normal reaction force N:



Taking components of the weight gives $mg\cos\theta - N = \frac{mv^2}{R}$. ✓

Hence $N = mg\cos\theta - \frac{mv^2}{R}$. ✓

Substituting the expression for the speed from above gives $N = mg\cos\theta - 2mgR(1-\cos\theta)$. ✓

And the result $N = mg(3\cos\theta - 2)$ follows.

c The marble will lose contact when $N \to 0$, i.e. when $\cos\theta = \frac{2}{3}$ or $\theta \approx 48°$. ✓

15 a Calling this distance x we have that:

$\frac{G16M}{x^2} = \frac{GM}{(d-x)^2}$ ✓

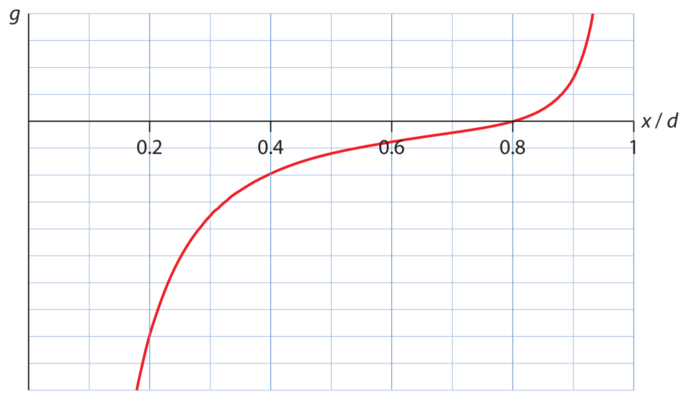
$16(d-x)^2 = x^2$ or $4(d-x) = \pm x$ ✓

Only the plus sign gives a positive distance and so $x = \frac{4d}{5}$. ✓

b Correct sign. ✓

Correct intersection. ✓

(The negative of this graph is also acceptable)

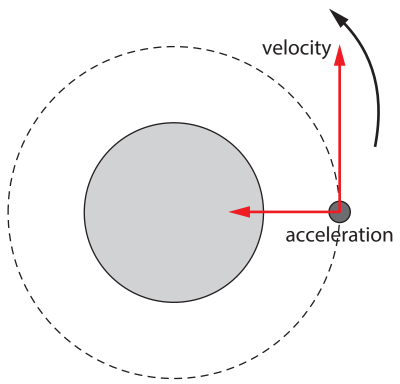


c **i** The force is zero. ✓

ii The force from the larger mass will be larger because the particle will be closer to it. ✓
Hence the net force will be directed towards the large mass. ✓

d It will move to the left. ✓
With increasing speed and increasing acceleration. ✓

16 **a** **i** Velocity arrow. ✓
Acceleration arrow. ✓



ii Acceleration is the rate of change of the velocity vector. ✓
Here the velocity vector is changing because its direction is so we have acceleration. ✓

b The force on the satellite is $\frac{GMm}{r^2} = m\frac{v^2}{r}$ i.e. $\frac{GM}{r} = v^2$. ✓

Using $v = \omega r$, ✓

gives $\frac{GM}{r} = \omega^2 r^2$. ✓

From which the result $\omega^2 r^3 = GM$ follows.

c **i** Since r decreases, from $\omega^2 r^3 = GM$ the angular speed will increase. ✓

ii From $\frac{GM}{r} = v^2$, as r decrease v increases. ✓

d **i** Using $\omega^2 r^3 = GM$ we find $M = \frac{\omega^2 r^3}{G}$ ✓

And so $M = \frac{(5.31\times 10^{-5})^2 \times (2.38\times 10^8)^3}{6.67\times 10^{-11}} = 5.70\times 10^{26}$ kg. ✓

ii Again using $\omega^2 r^3 = GM$ we find $\omega_T^2 r_T^3 = \omega_E^2 r_E^3$. ✓

Hence $\omega_T = \omega_E\sqrt{\frac{r_E^3}{r_T^3}} = 5.31\times 10^{-5} \times \sqrt{\left(\frac{2.38\times 10^8}{1.22\times 10^9}\right)^3} = 4.58\times 10^{-6}\ \text{rad s}^{-1}$ ✓

Hence $T = \frac{2\pi}{\omega_T} = \frac{2\pi}{4.58\times 10^{-6}} = 1.37\times 10^6\ \text{s} = \frac{1.37\times 10^6}{24\times 3600}\ \text{d} = 15.856 \approx 15.9\ \text{d}$ ✓